been increasing: in this sense, there might be a link running from income distribution to market structure, consistent with the observed data.

Two final points should be noticed: first, we treat income polarisation as an exogenous shock; secondly, we do not take into account possible income increases. Exogeneity of the income distribution is consistent with our partial equilibrium approach, in the sense that we assume away any feedback effects from market concentration to aggregate income distribution. As far as the second point is concerned, in the real world income polarization has been associated with increases in average income; however, we abstract from the latter and focus on mean-preserving shocks to income distribution, in order to sort out the effects of purely distributive changes.

The outline of the paper is as follows: section 2 presents the basics of the model; section 3 performs comparative statics exercises, taking into account the effects of income polarisation; comments and conclusions are gathered in section 4.

2 The basic model

We consider, in turn, (i) the demand side, describing the income distribution, the optimal decision of consumers, and the resulting market demand function; (ii) the optimal decision of symmetric firms in an oligopoly setting *à la* Cournot.

2.1 Income distribution and demand

We model the demand side of the market as a *continuum* of consumers, each of whom is identified by the income y he is endowed with.The latter is continuously distributed as $F : [y_{\min}, y_{\max}] \rightarrow [0, 1]$ over some support such that $0 \leq y_{\min} < y_{\max}$. The *only* assumptions we impose on F (apart from differentiability) are that (a) the density $f(y, \theta) = \partial F(y, \theta)/\partial y$ is unimodal; (b) it is subject to mean preserving shocks – i.e., if we take a real parameter θ as a mean preserving spread, an increase in θ translates itself into the distribution $f(\cdot, \theta)$ being more dispersed around the given mean.[3] If we denote the interior mode by $m \in (y_{\min}, y_{\max})$, we can write formally (a) and (b) as the following properties:

[3]Using a mean preserving spread amounts to ranking equal-mean distributions by second-order stochastic dominance. It is well known that such ranking is equivalent to Lorenz dominance: θ is thus an inequality index satisfying the Pigou-Dalton's "principle of transfers" (Atkinson, 1970).

$$
\begin{cases}
\dfrac{\partial f(m,\theta)}{\partial y} = 0 \\
\dfrac{\partial f(y,\theta)}{\partial y} > 0 \text{ for } y < m \\
\dfrac{\partial f(y,\theta)}{\partial y} < 0 \text{ for } y > m
\end{cases}
\tag{a}
$$

$$
\begin{cases}
\displaystyle\int_{y_{\min}}^{y} \frac{\partial F(x,\theta)}{\partial \theta} dx \geq 0, \qquad y < y_{\max} \\
\displaystyle\int_{y_{\min}}^{y_{\max}} \frac{\partial F(x,\theta)}{\partial \theta} dx = 0
\end{cases}
\tag{b}
$$

These properties yield a useful result, summarized in the following proposition:

Proposition 1 *If θ is a mean preserving spread of the distribution $F(y,\theta)$, then:*
(i) there exists a value $\widehat{y} \in (y_{\min}, y_{\max})$ such that $\frac{\partial F(y,\theta)}{\partial \theta} \geq 0$ for all $y \in (y_{\min}, \widehat{y})$, with strict inequality somewhere;
(ii) there exists a value $\overline{y} < \widehat{y}$ such that $\frac{\partial f(y,\theta)}{\partial \theta} \geq 0$ for all $y \in (y_{\min}, \overline{y})$, with strict inequality somewhere.

Proof. For ease of notation, let $G(y,\theta) \equiv \frac{\partial F(y,\theta)}{\partial \theta}$. Then the following holds: (1) $G(y_{\min},\theta) = G(y_{\max},\theta) = 0$, which follows from θ not altering the distribution's range; (2) $\exists \widehat{y}$ such that $G(y,\theta) \geq 0$ for all $y < \widehat{y}$, with $\widehat{y} > y_{\min}$ and strict inequality somewhere, since by the definition of mean preserving spread (property (b)) the integral cannot be negative around $y_{\min}$. This proves claim *(i);* (3) $G(y,\theta) < 0$ for some $y > \widehat{y}$, by the same property, as the integral function is zero around $y_{\max}$. All of this implies that $G(y,\theta)$ crosses zero at least once in the interior of $[y_{\min}, y_{\max}]$, the first time at $\widehat{y}$ from above. Hence G exhibits a local maximum $\overline{y}$, between $y_{\min}$ and $\widehat{y}$. There follows that $\frac{\partial G(y,\theta)}{\partial y} \geq 0$ for all $y \in (y_{\min}, \ \overline{y})$ (with strict inequality somewhere). Then claim (ii) follows trivially, since by Young's theorem on cross derivatives $\frac{\partial G(y,\theta)}{\partial y} = \frac{\partial f(y,\theta)}{\partial \theta}$. ■

The basic idea is illustrated by the following figure, where the simple case of single crossing of the distribution is described (it should however be stressed that we do *not* impose single crossing): there is a value $\overline{y}$ such that for all $y \leq \overline{y}$ both the density and the distribution are raised by an increase of the mean preserving spread θ. Notice that $\overline{y}$ might not be arbitrarily close to $y_{\min}$, as is apparent from Figure 1.[4]

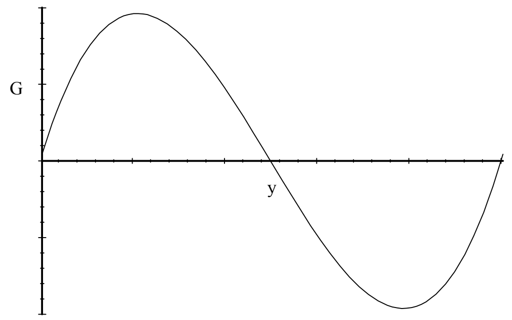

Figure 1: The function G

We assume that any given consumer buys one unit of the commodity whenever its price p is lower than his 'income' y – which of course amounts to interpreting y as the consumer's reservation price.[5] If one normalizes to unity the total population, market demand is then simply

$$Q(p,\theta) = 1 - F(p,\theta) \tag{1}$$

the (positive) elasticity of which can straightforwardly be derived as $\eta(p,\theta) = pf(p,\theta)/[1 - F(p,\theta)]$.

2.2 The Cournot equilibrium

The supply side of the model is described by a symmetric Cournot setup, with non-decreasing marginal and average variable costs, and non-negative fixed costs. Assume then that the market is served by n identical firms, and let $C(q_i)$, $i = 1, ..., n$, denote variable costs and $C_M(q_i) = C(q_i)/q_i$ average variable costs: we have $C'(q_i) \geq C_M(q_i) \geq 0$, and $C''(q_i) \geq 0$ for $q_i \geq 0$. We also assume $C'(0) \in [y_{\min}, y_{\max})$.

[4] Consider e.g. a symmetric Beta distribution $f(y,\theta) = y^\theta(1-y)^\theta/B(\theta)$, with $\theta > 0$, $y \in [0,1]$ and $B(\theta) = \int_0^1 y^\theta(1-y)^\theta dy$. Then a decrease in θ is a mean preserving spread in the distribution, and $\theta = 1$ (quadratic distribution) yields $\overline{y} \cong 0.253$ – that is, around the 25th percentile of the distribution.

[5] This is clearly the most direct way to link reservation prices to income. Since we are assuming no specific functional form for income distribution, our argument only requires that a unimodal income distribution generates a unimodal distribution of reservation prices, and that a wider spread in income distribution be mirrored into a wider spread of the distribution of reservation prices.

Given the market demand function (1), firm i maximizes its profits. In this setting it is easier to solve the Cournot model in prices along the lines suggested, e.g., by Kreps (1990, ch.10). This entails that the individual demand curve faced by firm i may be written as

$$q_i(p_i, p_{-i}; \theta) = 1 - F(p_i, \theta) - \sum_{j \neq i} q_j(p_j, p_{-j}; \theta)$$

where $p_{-i} = \{p_j\}_{j \neq i}$. The function to be maximized is

$$\pi_i(p_i, p_{-i}; K, \theta) = q_i(p_i, p_{-i}; \theta)p_i - C(q_i(p_i, p_{-i}; \theta)) - K \tag{2}$$

where $K \geq 0$ denotes fixed costs, and the Cournot conjecture entertained by firm i is $\partial q_j / \partial p_i = 0$. Firm i's first order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = 1 - F(p_i, \theta) - [p_i - C'(\cdot)]f(p_i, \theta) - \sum_{j \neq i} q_j = 0$$

Invoking symmetry, $p_i = p_j = p$ for all $i, j = 1, ..., n$, and hence

$$\sum_{j \neq i} q_j = \tfrac{n-1}{n}(1 - F(p, \theta)),$$

we obtain that in equilibrium

$$\frac{\partial \pi_i}{\partial p_i}_{|p_i = p} = \tfrac{1}{n}(1 - F(p, \theta)) - [p - C'(\cdot)]f(p, \theta) = 0 \tag{3}$$

which of course amounts to the familiar Cournot condition $(1 - \frac{1}{\eta n})p = C'$. Equation (3) solves for the sought-for short-run equilibrium price $p^*(n, \theta)$.

Dropping the i subscript, the corresponding equilibrium profit of the generic firm is

$$\pi^e = \tfrac{1}{n}\left(1 - F(p^*, \theta)\right)p^* - C\left(\tfrac{1}{n}\left(1 - F(p^*, \theta)\right)\right) - K \tag{4}$$

Clearly, the equilibrium number of firms n^* is detemined by the zero profit condition $\pi^e = 0$, which yields an implicit function $n^*(K, \theta)$.[6] In the Appendix we show that, given our assumptions on costs and demand, one such equilibrium exists and exhibits the (standard) properties summarized in the following:

[6] We treat n as a continuous variable, following a well established practice (e.g., Mankiw and Whinston, 1986).

Proposition 2 *Consider the normalized demand curve* $Q(p,\theta) = 1 - F(p,\theta)$*, where* $F(p,\theta)$ *is a unimodal income distribution and* θ *a mean preserving spread. Assume that (i) average variable costs* $C(q)/q$ *are nondecreasing in* q*; (ii) marginal costs* $C'(\cdot)$ *are non-decreasing in* q *and such that* $y_{\min} \leq C'(0) < y_{\max}$*. Then (a) the symmetric equilibrium price* $p^*(n,\theta)$ *obtained from (3) is monotonically decreasing in* n*, that is* $\frac{dp^*}{dn} < 0$*; and (b) the long run Cournot equilibrium price* $p^*(K,\theta) = p^*(n^*(K,\theta),\theta)$ *decreases monotonically to its perfect competition level as* K *tends to zero, that is* $\frac{dp^*}{dK} > 0$ *and* $\lim_{K\to 0} p^*(K,\theta) = \lim_{K\to 0} p^*(n^*(K,\theta),\theta) = C'(0)$*.*

Property (a) is known as 'quasi-competitiveness': it refers to industry output increasing as n increases. It should be noticed that property (b) (monotonic convergence to the competitive equilibrium) is not necessarily implied by the first (e.g., Ruffin, 1971).

3 Income distribution and the number of firms

The behaviour of the function $n^*(K,\theta)$ will tell us how the long-run equilibrium number of firms adjusts to changes in demand brought about by variations in θ, i.e. by mean-preserving increases in income dispersion. One can approach this problem by totally differentiating the zero profit condition. This gives

$$\frac{\partial \pi^e}{\partial p}\left(\frac{dp^*}{dn}dn + \frac{dp^*}{d\theta}d\theta\right) + \frac{\partial \pi^e}{\partial n}dn + \frac{\partial \pi^e}{\partial \theta}d\theta = 0 \tag{5}$$

Notice that the first term is not nil - that is, one cannot take advantage of the envelope theorem, since the firm does not maximize profit as defined by (4). There is an obvious externality involved, due to oligopolistic interaction: $\partial\pi^e/\partial p$ as derived from (4) is different from $\partial\pi_i/\partial p_i$ as defined in (3). The latter is clearly nil, while the former is not – indeed it is positive for $n > 1$, precisely because we know that $\partial\pi_i/\partial p_i = 0$: by comparing the two, it is easily checked that there is a factor $1/n$ of difference, such that the two collapse to the same (nil) value for $n = 1$ under monopoly, or under perfect competition as n tends to infinity - in both cases there is no externality.[7]

This being said, we are able to derive the paper's main result, to the effect that, if the fixed cost K is sufficiently low, shifting the mass of incomes towards the tails of the distribution *always* decreases the equilibrium number of firms surviving in the long-run.

[7] This is shown formally in the proof of Proposition 3, below.